New Tuning Conditions for a Class of Nonlinear PID Global Regulators of Robot Manipulators.

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***Abstract*— The motivation of this work relies on the fact that, until now, the tuning conditions presented in the literature for assuring global asymptotic stability are conservative in the sense that controller gains for joints with a little or nothing of gravitational torques are tuned in the same form that the joints which support large gravitational torques. In this paper we extend the more recent tuning conditions presented in the literature for a class of nonlinear PID global regulators of robot manipulators. Such an extension has permitted to carry out by the first time experimental essays for a class of nonlinear PID global regulators, which are presented in this paper using a two degrees of freedom robot manipulator.**

Keywords: robot manipulators, nonlinear PID, global stability, tuning conditions.

I. Introduction

The study of PID controllers in robot manipulators has been subject of extensive researches in many years over the past. Several works have been presented proving that PID controllers guarantee semiglobal asymptotically stability of the closed-loop equilibrium point in the case of set-point control (Arimoto, 1984), (Arimoto and Suzuki, 1990), (Kelly, 1995), (Ortega *et al.*, 1995), (Alvarez-Ramirez *et al.*, 2000), (Meza *et al.*, 2007), (Hernandez *et al.*, 2008). Due to the classical linear PID controllers has only been proved to be semiglobally asymptotically stable, several nonlinear PID global controllers have been proposed in some works (Arimoto, 1995), (Kelly, 1998), (Santibañez and Kelly, 1998). These nonlinear PID controllers, unlike the linear PID controller, yield global asymptotical stability of the closed-loop equilibrium point. In (Santibañez and Kelly, 1998) it was proposed a class of nonlinear PID global regulators for robot manipulators, which encompasses the particular cases of (Arimoto, 1995) and (Kelly, 1998).

However, an important drawback exists: the tuning conditions for assuring global asymptotic stability for the aforementioned nonlinear PID controllers are far from being acceptable in practical applications. The latter in the sense that the tuning conditions previously reported are conservative and overestimated because such conditions are established as expressions of norms of gain parameter vectors and matrices. This make that all the joint gains be restricted by the same rule, in such a way that the conditions for larger and smaller joints is the same. This paper, inspired in (Hernandez-Guzman *et al.*, 2008) and (Hernandez-Guzman and Silva-Ortigoza, 2011), addresses a tuning procedure which allows to obtain conditions less restrictive for joints in order to get better performances. Such tuning procedure has permitted to carry out experimental essays which are reported in this work using a two degrees of freedom robot manipulator.

Finally, some remarks on notation. Given some vector $\boldsymbol{x} \in \mathbb{R}^n$ and some matrix $A(\boldsymbol{x}) \in \mathbb{R}^{n\times n}$, the Euclidean norm of $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}}$ and the spectral norm of $A(\boldsymbol{x})$ is defined as $\|A(\boldsymbol{x})\| = \sqrt{\lambda_{\max}(A^TA)}$ where $\lambda_{\max}(A^TA)$ is the largest eigenvalue of the symmetric matrix A^TA. In the case where $A(\boldsymbol{x}) \in \mathbb{R}^{n\times n}$ is a symmetric matrix, then $\|A\| = \max_i |\lambda_i(A)|$, where $\lambda_i(A)$ and $|\cdot|$ are eigenvalues of $A(\boldsymbol{x})$ and the absolute value function, respectively. $\lambda_{\min}$ stands for the smallest eigenvalue of A for all $\boldsymbol{x} \in \mathbb{R}^n$. We use symbol y_i to represent the i–th component if $\boldsymbol{y}$ is a vector or the i–th diagonal entry if Y is a diagonal matrix.

II. Preliminaries

A. Robot Dynamics

Consider the dynamic model for a serial n–link rigid robot, given by

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + F_v\dot{\boldsymbol{q}} = \boldsymbol{\tau} \qquad (1)$$

where $\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q} \in \mathbb{R}^n$ are the vectors of joint accelerations, joint velocities and joint positions, respectively, $\tau \in \mathbb{R}^n$ is the vector of applied torques, $M(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is the symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \mathbb{R}^{n\times n}$ is the matrix of centripetal and Coriolis torques, $\boldsymbol{g}(\boldsymbol{q})$ is the vector of gravitational torques obtained as the gradient of the robot potential energy, i. e. $\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial\mathcal{U}(\boldsymbol{q})}{\partial\boldsymbol{q}}$, and $F_v \in \mathbb{R}^{n\times n}$ is a constant diagonal definite positive matrix which represent the viscous friction coefficient at each joint. We assume that all joints of the robot are of the revolute type.

B. Properties of the robot dynamics

In the following two properties of the dynamics (1) are presented.

Property 1. Matrices $M(\boldsymbol{q})$ and $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ satisfy:

$$\dot{\boldsymbol{q}}^T\left(\frac{1}{2}\dot{M}(\boldsymbol{q})-C(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}}=0 \tag{2}$$

$$\dot{M}(\boldsymbol{q})=C(\boldsymbol{q},\dot{\boldsymbol{q}})+C^T(\boldsymbol{q},\dot{\boldsymbol{q}}). \tag{3}$$

Furthermore, there exists a positive constant k_{c_1} such that, for all $\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}\in\mathbb{R}^n$:

$$\|C(\boldsymbol{x},\boldsymbol{y})\boldsymbol{z}\|\le k_{c_1}\|\boldsymbol{y}\|\|\boldsymbol{z}\|. \tag{4}$$

Property 2. The gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$ is bounded for all $\boldsymbol{q}\in\mathbb{R}^n$. This means that there exist finite constants $k_i'\ge 0$ such that:

$$\sup_{\boldsymbol{q}\in\mathbb{R}^n}|g_i(\boldsymbol{q})|\le k_i' \qquad i=1,\cdots,n, \tag{5}$$

where $g_i(\boldsymbol{q})$ stands for the elements of $\boldsymbol{g}(\boldsymbol{q})$. Equivalently, there exists a constant k' such that $\|\boldsymbol{g}(\boldsymbol{q})\|\le k'$ for all $\boldsymbol{q}\in\mathbb{R}^n$.
Furthermore there exists a positive constant k_{g_i} such that

$$\sum_{j=1}^{n}\max_{\boldsymbol{q}}\left|\frac{\partial g_i(\boldsymbol{q})}{\partial q_j}\right|\le k_{g_i},$$

for all $\boldsymbol{q}\in\mathbb{R}^n$ and $i=1,2,...,n$.

Finally, we present some useful results.

Theorem 1: (Kelly *et al.*, 2005) Let $f:\mathbb{R}^n\to\mathbb{R}$ be a continuously differentiable function with continuous partial derivatives up to at least second order. Assume that:

$$f(\mathbf{0})=0\in\mathbb{R} \tag{6}$$

$$\frac{\partial f}{\partial \boldsymbol{x}}(\mathbf{0})=0\in\mathbb{R}^n \tag{7}$$

If the Hessian matrix $\partial/\partial\boldsymbol{x}[\partial f(\boldsymbol{x})/\partial\boldsymbol{x}]$ is positive definite for all $\boldsymbol{x}\in\mathbb{R}^n$, then $f(\boldsymbol{x})$ is a globally positive definite and radially unbounded function.

Theorem 2: (Kelly *et al.*, 2005) (*Mean value theorem for vectorial functions*). Consider the continuous vectorial function $\boldsymbol{f}:\mathbb{R}^n\to\mathbb{R}^m$. If $\boldsymbol{f}_i(z_1,z_2,...,z_n)$ has continuous partial derivatives for $i=1,...,m$, then for each pair of vectors $\boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^n$ and each $\boldsymbol{w}\in\mathbb{R}^m$ there exists $\boldsymbol{\xi}\in\mathbb{R}^n$ such that:

$$[\boldsymbol{f}(\boldsymbol{x})-\boldsymbol{f}(\boldsymbol{y})]^T\boldsymbol{w}=\boldsymbol{w}^T\left.\frac{\partial\boldsymbol{f}(\boldsymbol{z})}{\partial(\boldsymbol{z})}\right|_{\boldsymbol{z}=\boldsymbol{\xi}}(\boldsymbol{x}-\boldsymbol{y}) \tag{8}$$

where $\boldsymbol{\xi}$ is a vector on the line segment that joins the vector $\boldsymbol{x}$ and $\boldsymbol{y}$.

Definition 1: Let A be a $n\times n$ matrix with a_{ij} representing its element at row i and column j. The matrix A is said to be strictly diagonally dominant if:

$$|a_{ii}|>\sum_{j=1,j\neq i}^{n}|a_{ij}|,\quad i=1,...,n. \tag{9}$$

Definition 2: If A is a $n\times n$ symetric and strictly diagonally dominant matrix and if $a_{ii}>0$ for all $i=1,...,n$, then A is positive definite.

Definition 3: $\mathcal{F}(k_a,\varepsilon,\boldsymbol{x})$ with $1\ge k_a>0$, $\varepsilon>0$, and $\boldsymbol{x}\in\mathbb{R}^n$ denotes the set of all continuous differentiable increasing functions $\mathbf{sat}(\boldsymbol{x})=[\mathrm{sat}(x_1)\quad\mathrm{sat}(x_2)\ldots\mathrm{sat}(x_n)]^T$ such that

- $\mathrm{sat}(-x)=-\mathrm{sat}(x)$
- $\mathrm{sgn}(x)=\mathrm{sgn}(\mathrm{sat}(x))$
- $|x|\ge|\mathrm{sat}(x)|\ge k_a|x|,\quad\forall\, x\in\mathbb{R}:|x|<\varepsilon$
- $\varepsilon\ge|\mathrm{sat}(x)|\ge k_a\varepsilon,\quad\forall\, x\in\mathbb{R}:|x|\ge\varepsilon$
- $1\ge(d/dx)\mathrm{sat}(x)\ge 0$
- $\|\mathbf{sat}(\boldsymbol{x})\|\ge\begin{cases}k_a\|\boldsymbol{x}\|, & \text{if }\ \|\boldsymbol{x}\|<\varepsilon\\ k_a\varepsilon, & \text{if }\ \|\boldsymbol{x}\|\ge\varepsilon\end{cases}$

Definition 4: Given positive constants l and m, with $l<m$, a function $\mathrm{Sat}(x;l,m):\mathbb{R}\to\mathbb{R}:x\mapsto\mathrm{Sat}(x;l,m)$ is said to be a strictly increasing linear saturation function for (l,m) if it is locally Lipschitz, strictly increasing, C^2 differentiable and satisfies:

1) $\mathrm{Sat}(x;l,m)=x$ when $|x|\le l$
2) $|\mathrm{Sat}(x;l,m)|<m$ for all $x\in\mathbb{R}$.

Lemma 1: Let $A\in\mathbb{R}^n$ be a matrix with a_{ij} representing the element in row i and column j, y $\boldsymbol{x}$, $\mathbf{sat}(\boldsymbol{x})$ two vectors such that:

$$\boldsymbol{x}=\begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix},\quad\mathbf{sat}(\boldsymbol{x})=\begin{bmatrix}\mathrm{sat}(x_1)\\\mathrm{sat}(x_2)\\\vdots\\\mathrm{sat}(x_n)\end{bmatrix} \tag{10}$$

where $\mathrm{sat}(\cdot)$ is a function like that described in Definition 3. If A is symmetric and strictly diagonally dominant, namely

$$|a_{ii}|>\sum_{j=1,j\neq i}^{n}|a_{ij}|,\quad i=1,2,...,n, \tag{11}$$

and $a_{ii}>0$ for all $i=1,2,...,n$, then

$$\mathbf{sat}(\boldsymbol{x})^TA\boldsymbol{x}>0,\quad\forall\quad\boldsymbol{x}\in\mathbb{R}^n,\quad\text{with }\ \boldsymbol{x}\neq\mathbf{0}\in\mathbb{R}^n;$$

i.e., $\mathbf{sat}(\boldsymbol{x})^TA\boldsymbol{x}$ is a positive definite continuous function.
Outline of the proof: The proof is based in the Sylvester Theorem proof by using the following nice property of the saturation function $\mathrm{sat}(x)\in\mathcal{F}(k_a,\varepsilon,x)$:

$$(x_i+x_j)(\mathrm{sat}(x_i)+\mathrm{sat}(x_j))\ge 0,$$
$$(x_i-x_j)(\mathrm{sat}(x_i)-\mathrm{sat}(x_j))\ge 0,\quad\forall\quad x_i,x_j\in\mathbb{R}$$

III. MAIN RESULT

A. A class of nonlinear PID controllers

Consider a class of nonlinear PID controllers like that presented in (Santibañez and Kelly, 1998), which can be written as:

$$\begin{aligned}\boldsymbol{\tau} &= \nabla_{\tilde{\boldsymbol{q}}}\mathcal{U}_a(\tilde{\boldsymbol{q}})-K_v\dot{\boldsymbol{q}}+K_i\boldsymbol{w}\\ \boldsymbol{w}(t) &= \int_0^t[\alpha\mathbf{sat}(\tilde{\boldsymbol{q}}(\sigma))-\dot{\boldsymbol{q}}(\sigma)]d\sigma+\boldsymbol{w}(0)\end{aligned} \tag{12}$$

where K_v and K_i are diagonal positive definite $n\times n$ matrices, $\tilde{\boldsymbol{q}}=\boldsymbol{q}_d-\boldsymbol{q}$ denotes the position error vector, $\alpha>0$ is a constant scalar, $\mathcal{U}_a(\tilde{\boldsymbol{q}})$ is a kind of C^1 artificial potential energy induced by a part of the proportional term of the controller whose properties were established in (Santibañez and Kelly, 1998).

Two examples of this kind of nonlinear PID regulators are:

- Controller presented in (Kelly, 1998)

$$\begin{aligned}\boldsymbol{\tau} &= K_p\tilde{\boldsymbol{q}}-K_v\dot{\boldsymbol{q}}+K_i\boldsymbol{w}\\ \boldsymbol{w}(t) &= \int_0^t[\alpha\mathbf{sat}(\tilde{\boldsymbol{q}}(\sigma))-\dot{\boldsymbol{q}}(\sigma)]d\sigma+\boldsymbol{w}(0)\end{aligned} \tag{13}$$

This controller has associated an artificial potential energy $\mathcal{U}_a(\tilde{\boldsymbol{q}})$ given by

$$\mathcal{U}_a(\tilde{\boldsymbol{q}}) = \frac{1}{2}\tilde{\boldsymbol{q}}^T K_p \tilde{\boldsymbol{q}} \tag{14}$$

- Controller presented in (Santibañez and Kelly, 1998)

$$\begin{aligned} \boldsymbol{\tau} &= K_p \mathbf{sat}(\tilde{\boldsymbol{q}}) - K_v \dot{\boldsymbol{q}} + K_i' \boldsymbol{w} \qquad (15) \\ \boldsymbol{w}(t) &= \int_0^t [\alpha \mathbf{sat}(\tilde{\boldsymbol{q}}(\sigma)) - \dot{\boldsymbol{q}}(\sigma)] d\sigma + \boldsymbol{w}(0) \end{aligned}$$

This controller has associated an artificial potential energy $\mathcal{U}_a(\tilde{\boldsymbol{q}})$ given by

$$\mathcal{U}_a(\tilde{\boldsymbol{q}}) = \int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}} K_p \mathbf{sat}(\boldsymbol{r}) d\boldsymbol{r} := \sum_{i=1}^{n} \int_0^{\tilde{q}_i} k_{pi} \text{sat}(r_i) dr_i \tag{16}$$

A particular case of this control law was presented in (Arimoto, 1995), with

$$\text{sat}(x) = \text{Sin}(x) = \begin{cases} \sin(x) & \text{for } |x| \le \frac{\pi}{2} \\ 1 & \text{for } x > \frac{\pi}{2} \\ -1 & \text{for } x < -\frac{\pi}{2}. \end{cases}$$

The stability conditions for the latter nonlinear PID controllers presented in (Santibañez and Kelly, 1998), as well as in (Kelly, 1998) for the first nonlinear PID controller and (Arimoto, 1995) for the second one, are very conservative in the sense that are far from being acceptable in practical applications, specially when they are used in robot manipulators with many degrees of freedom. In the following a new tuning procedure which allows to obtain stability conditions less restrictive to assure global asymptotic stability is proposed.

B. Stability analysis (New tuning conditions)

Inspired in (Hernandez-Guzman *et al.*, 2008) and (Hernandez-Guzman and Silva-Ortigoza, 2011), in this section we present new tuning conditions for the stability analysis of the class of nonlinear PID controllers introduced in (Santibañez and Kelly, 1998).

B.1. Consider the control law (13):

$$\begin{aligned} \boldsymbol{\tau} &= K_p \tilde{\boldsymbol{q}} - K_v \dot{\boldsymbol{q}} + K_i \boldsymbol{w} \\ \boldsymbol{w} &= \int_0^t [\alpha \mathbf{sat}(\tilde{\boldsymbol{q}}(\sigma)) - \dot{\boldsymbol{q}}(\sigma)] d\sigma + \boldsymbol{w}(0), \end{aligned}$$

which was proposed in (Kelly, 1998) and reanalyzed in (Hernandez-Guzman and Silva-Ortigoza, 2011). This control law has been rewritten to express it like the equation (12). K_p, K_v and K_i are $n \times n$ diagonal positive definite matrices, $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the position error vector and $\alpha > 0$ is a small constant suitably selected.

By using the control law (13) into the dynamics (1), we obtain the following closed-loop system

$$\frac{d}{dt}\begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \\ \boldsymbol{z} \end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}[K_p\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}} + K_i\boldsymbol{z} \\ -C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - F_v\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{g}(\boldsymbol{q}_d)] \\ \alpha\mathbf{sat}(\tilde{\boldsymbol{q}}) - \dot{\boldsymbol{q}} \end{bmatrix} \tag{17}$$

where $\boldsymbol{z}$ is defined as

$$\boldsymbol{z} = \int_0^t [\alpha \mathbf{sat}(\tilde{\boldsymbol{q}}(\sigma)) - \dot{\boldsymbol{q}}(\sigma)] d\sigma + \underbrace{\boldsymbol{w}(0) - K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d)}_{\boldsymbol{z}(0)} \tag{18}$$

so that (17) is an autonomous differential equation whose unique equilibrium is:

$$[\tilde{\boldsymbol{q}}^T \quad \dot{\boldsymbol{q}}^T \quad \boldsymbol{z}^T]^T = \mathbf{0} \in \mathbb{R}^{3n}. \tag{19}$$

Proposition 1. There always exist a positive scalar constant α and positive definite matrices K_p, K_v, $K_i \in \mathbb{R}^{n\times n}$ such that the equilibrium point (19) from (17) is globally asymptotically stable.

Proof. Based on the Lyapunov theory, we propose the following Lyapunov function candidate

$$V(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{z}) = V_1(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{z}) + V_2(\tilde{\boldsymbol{q}}) + V_3(\tilde{\boldsymbol{q}}) \tag{20}$$

where

$$\begin{aligned} V_1 &= \frac{1}{2}[\dot{\boldsymbol{q}} - \alpha\mathbf{sat}(\tilde{\boldsymbol{q}})]^T M(\boldsymbol{q})[\dot{\boldsymbol{q}} - \alpha\mathbf{sat}(\tilde{\boldsymbol{q}})] + \frac{1}{2}\boldsymbol{z}^T K_i \boldsymbol{z} \\ &\quad + \sum_{i=1}^{n}\int_0^{\tilde{q}_i} \alpha f_{v_i}\text{sat}(r_i)dr_i + \sum_{i=1}^{n}\int_0^{\tilde{q}_i} \alpha k_{v_i}\text{sat}(r_i)dr_i \\ V_2 &= \frac{1}{2}\tilde{\boldsymbol{q}}^T\overline{K_p}\tilde{\boldsymbol{q}} - \frac{\alpha^2}{2}\mathbf{sat}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\mathbf{sat}(\tilde{\boldsymbol{q}}) \\ V_3 &= \mathcal{U}(\boldsymbol{q}_d - \tilde{\boldsymbol{q}}) - \mathcal{U}(\boldsymbol{q}_d) + \frac{1}{2}\tilde{\boldsymbol{q}}^T K_p^* \tilde{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}}, \end{aligned}$$

where $K_p = \overline{K_p} + K_p^*$. Thus, (20) will be a radially unbounded positive definite function provided that V_2 and V_3 be also a radially unbounded positive definite function. To this end, we provide lower bounds on the following terms:

$$\frac{1}{2}\tilde{\boldsymbol{q}}^T\overline{K_p}\tilde{\boldsymbol{q}} = \frac{1}{2}\sum_{i=1}^{n}\overline{k_{p_i}}\tilde{q}_i^2 \tag{21}$$

$$\begin{aligned} -\frac{\alpha^2}{2}\mathbf{sat}^T(\tilde{\boldsymbol{q}})M(\boldsymbol{q})\mathbf{sat}(\tilde{\boldsymbol{q}}) &\ge -\sum_{i=1}^{n}\frac{\alpha^2}{2}\lambda_{\max}\{M(\boldsymbol{q})\}\text{sat}^2(\tilde{q}_i) \\ &\ge -\sum_{i=1}^{n}\frac{\alpha^2}{2}\lambda_{\max}\{M(\boldsymbol{q})\}H(\tilde{q}_i) \end{aligned} \tag{22}$$

$$H(\tilde{q}_i) = \begin{cases} \tilde{q}_i^2, & |\tilde{q}_i| \le \varepsilon \\ \varepsilon^2, & |\tilde{q}_i| > \varepsilon \end{cases}$$

From (21) and (22) we can conclude that there always exist large enough positive constants $\overline{k_{p_i}}$, $i = 1, ..., n$, such that:

$$\frac{1}{2}\overline{k_{p_i}}\tilde{q}_i^2 > \frac{\alpha^2}{2}\lambda_{\max}\{M(\boldsymbol{q})\}H(\tilde{q}_i) \quad \forall\ \tilde{\boldsymbol{q}} \neq \mathbf{0} \in \mathbb{R}^n \tag{23}$$

and hence

$$V_2 = \frac{1}{2}\tilde{\boldsymbol{q}}^T\overline{K_p}\tilde{\boldsymbol{q}} - \frac{\alpha^2}{2}\mathbf{sat}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\mathbf{sat}(\tilde{\boldsymbol{q}}) > 0 \quad \forall\ \tilde{\boldsymbol{q}} \neq \mathbf{0} \in \mathbb{R}^n$$

On the other hand, according to Theorem 1, and Definition 1 and 2, V_3 is definite positive if (Hernandez-Guzman *et al.*, 2008)

$$k_{p_i}^* > k_{g_i} \tag{24}$$

Finally, we can conclude that (20) is definite positive and radially unbounded if (23) and (24) are satisfied.

It is possible to verify that, using Property 1, the time derivative of (20) along the trajectories of the closed-loop system (17) is given by:

$$\begin{aligned} \dot{V} &= -\dot{\boldsymbol{q}}^T F_v \dot{\boldsymbol{q}} - \alpha\dot{\mathbf{sat}}(\tilde{\boldsymbol{q}})M(\boldsymbol{q})\dot{\boldsymbol{q}} - \alpha\mathbf{sat}(\tilde{\boldsymbol{q}})^T C(\boldsymbol{q},\dot{\boldsymbol{q}})^T\dot{\boldsymbol{q}} \\ &\quad - \alpha\mathbf{sat}(\tilde{\boldsymbol{q}})^T[K_p\tilde{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{g}(\boldsymbol{q})] - \dot{\boldsymbol{q}}^T K_v \dot{\boldsymbol{q}}. \end{aligned} \tag{25}$$

Negative semidefiniteness may be proved by upper bounding each element of (25):

$$\begin{aligned}
-\dot{\boldsymbol{q}}^T F_v \dot{\boldsymbol{q}} &\le -\lambda_{\min}\{F_v\}\|\dot{\boldsymbol{q}}\|^2 \\
-\dot{\boldsymbol{q}}^T K_v \dot{\boldsymbol{q}} &\le -\lambda_{\min}\{K_v\}\|\dot{\boldsymbol{q}}\|^2 \\
-\alpha \mathbf{sat}(\tilde{\boldsymbol{q}})^T C(\boldsymbol{q},\dot{\boldsymbol{q}})^T \dot{\boldsymbol{q}} &\le \alpha\varepsilon k_c \|\dot{\boldsymbol{q}}\|^2 \\
-\alpha \dot{\mathbf{sat}}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} &\le \alpha\lambda_{\max}\{M\}\|\dot{\boldsymbol{q}}\|^2
\end{aligned}$$

and using Theorem 2, we have

$$-\alpha \mathbf{sat}(\tilde{\boldsymbol{q}})^T[\boldsymbol{g}(\boldsymbol{q}_d)-\boldsymbol{g}(\boldsymbol{q})] = -\alpha \mathbf{sat}(\tilde{\boldsymbol{q}})^T \left.\frac{\partial \boldsymbol{g}(\boldsymbol{z})}{\partial \boldsymbol{z}}\right|_{\boldsymbol{z}=\boldsymbol{\xi}} \tilde{\boldsymbol{q}}, \quad (26)$$

for some $\boldsymbol{\xi}$ belonging to the line that joins $\boldsymbol{q}_d$ and $\boldsymbol{q}$. Finally the time derivative $\dot{V}$ can be upper bounded by:

$$\begin{aligned}
\dot{V} \le\ & -[\lambda_{\min}\{F_v\}+\lambda_{\min}\{K_v\}-\alpha(\varepsilon k_c+\lambda_{\max}\{M\})]\|\dot{\boldsymbol{q}}\|^2 \\
& -\alpha \mathbf{sat}(\tilde{\boldsymbol{q}})^T \left[K_p + \left.\frac{\partial \boldsymbol{g}(\boldsymbol{z})}{\partial \boldsymbol{z}}\right|_{\boldsymbol{z}=\boldsymbol{\xi}}\right]\tilde{\boldsymbol{q}} \quad (27)
\end{aligned}$$

By using Definition 2, Definition 3 and Lemma 1, we can conclude that $\alpha \mathbf{sat}(\tilde{\boldsymbol{q}})^T \left[K_p + \left.\frac{\partial \boldsymbol{g}(\boldsymbol{z})}{\partial \boldsymbol{z}}\right|_{\boldsymbol{z}=\boldsymbol{\xi}}\right]\tilde{\boldsymbol{q}}$ is definite positive if the matrix $\left[K_p + \left.\frac{\partial \boldsymbol{g}(\boldsymbol{z})}{\partial \boldsymbol{z}}\right|_{\boldsymbol{z}=\boldsymbol{\xi}}\right]$ fulfills

$$k_{p_i} > k_{g_i}; \quad (28)$$

and by choosing a small enough value for $\alpha > 0$ such that

$$[\lambda_{\min}\{F_v\}+\lambda_{\min}\{K_v\}-\alpha(\varepsilon k_c+\lambda_{\max}\{M\})] > 0, \quad (29)$$

we have that (25) is negative semidefinite. Therefore, we can use the LaSalle invariance principle to assure global asymptotic stability provided that (23), (24) and (29) are satisfied. This completes the proof of Proposition 1.

B.2. Consider a control law similar to that presented in (Arimoto, 1995) and (Santibañez and Kelly, 1998):

$$\begin{aligned}
\boldsymbol{\tau} &= K_p \mathbf{Sat}(B\tilde{\boldsymbol{q}};l,m) - K_v\dot{\boldsymbol{q}} + K_i\boldsymbol{w} \quad (30) \\
\boldsymbol{w} &= \int_0^t [\alpha \mathbf{Sat}(B\tilde{\boldsymbol{q}}(\sigma);l,m) - \dot{\boldsymbol{q}}(\sigma)]d\sigma + \boldsymbol{w}(0),
\end{aligned}$$

where now, $\mathrm{Sat}(\cdot)$ is defined in Definition 4 and we have used a new gain $B = \mathrm{diag}\{\beta_1, \beta_2, ..., \beta_n\}$ multiplying $\tilde{\boldsymbol{q}}$ inside the saturation function. B, K_p, K_v and K_i are $n \times n$ diagonal positive definite matrices, l and m are the saturation limits and $\alpha > 0$ is a small constant suitably selected.
By using the control law (30) into the dynamics (1), we obtain the following closed-loop system

$$\frac{d}{dt}\begin{bmatrix}\tilde{\boldsymbol{q}}\\ \dot{\boldsymbol{q}}\\ \boldsymbol{z}\end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}[K_p\mathbf{Sat}(B\tilde{\boldsymbol{q}};l,m) - K_v\dot{\boldsymbol{q}} + K_i\boldsymbol{z} \\ -C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - F_v\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{g}(\boldsymbol{q}_d)] \\ \alpha\mathbf{Sat}(B\tilde{\boldsymbol{q}};l,m) - \dot{\boldsymbol{q}} \end{bmatrix} \quad (31)$$

where $\boldsymbol{z}$ is defined as

$$\boldsymbol{z} = \int_0^t [\alpha\mathbf{Sat}(B\tilde{\boldsymbol{q}}(\sigma);l,m) - \dot{\boldsymbol{q}}(\sigma)]d\sigma + \underbrace{\boldsymbol{w}(0) - K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d)}_{\boldsymbol{z}(0)} \quad (32)$$

so that (31) is an autonomous differential equation whose unique equilibrium is:

$$[\tilde{\boldsymbol{q}}^T \quad \dot{\boldsymbol{q}}^T \quad \boldsymbol{z}^T]^T = \mathbf{0} \in \mathbb{R}^{3n}. \quad (33)$$

From here, the saturation limits l and m are omitted for reasons of space.
Proposition 2. There always exist a positive scalar constant α and positive definite matrices B, K_p, K_v, $K_i \in \mathbb{R}^{n\times n}$ such that the equilibrium point (33) from (31) is globally asymptotically stable.
Proof. In base to Lyapunov theory, we propose the following Lyapunov function candidate

$$V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{z}) = V_1(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{z}) + V_2(\tilde{\boldsymbol{q}}) + V_3(\tilde{\boldsymbol{q}}) \quad (34)$$

where

$$\begin{aligned}
V_1 =\ & \frac{1}{2}[\dot{\boldsymbol{q}} - \alpha\mathbf{Sat}(B\tilde{\boldsymbol{q}})]^T M(\boldsymbol{q})[\dot{\boldsymbol{q}} - \alpha\mathbf{Sat}(B\tilde{\boldsymbol{q}})] + \frac{1}{2}\boldsymbol{z}^T K_i \boldsymbol{z} \\
& + \sum_{i=1}^n \int_0^{\tilde{q}_i} \alpha f_{v_i}\mathrm{Sat}(\beta_i r_i)dr_i + \sum_{i=1}^n \int_0^{\tilde{q}_i} \alpha k_{v_i}\mathrm{Sat}(\beta_i r_i)dr_i \\
V_2 =\ & \int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}} \overline{K_p}\mathbf{Sat}(B\boldsymbol{r})d\boldsymbol{r} - \frac{\alpha^2}{2}\mathbf{Sat}(B\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\mathbf{Sat}(B\tilde{\boldsymbol{q}}) \\
V_3 =\ & \int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}} [-\boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{r}) + K_p^*\mathbf{Sat}(B\boldsymbol{r};\boldsymbol{l},\boldsymbol{m}) + \boldsymbol{g}(\boldsymbol{q}_d)]^T d\boldsymbol{r}, \quad (35)
\end{aligned}$$

and $K_p = \overline{K_p} + K_p^*$. Thus, (34) will be a radially unbounded positive definite function provided that V_2 and V_3 be also a radially unbounded positive definite function.
Consider first V_2. Note that according to definition 4 and by direct integration we find that:

$$\int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}} \mathbf{Sat}^T(B\boldsymbol{r};l,m)\overline{K_p}dr = \sum_{i=1}^n G_i(\tilde{q}_i),$$

$$G_i(\tilde{q}_i) = \begin{cases} \frac{\overline{k_{p_i}}}{2}\beta_i\tilde{q}_i^2, & |\beta_i\tilde{q}_i| \le l \\ \frac{\overline{k_{p_i}}}{2}\frac{l^2}{\beta_i} + \overline{k_{p_i}}l(\tilde{q}_i - \frac{l}{\beta_i}), & \beta_i\tilde{q}_i > l \\ \frac{\overline{k_{p_i}}}{2}\frac{l^2}{\beta_i} - \overline{k_{p_i}}l(\tilde{q}_i + \frac{l}{\beta_i}), & \beta_i\tilde{q}_i < -l \end{cases} \quad (36)$$

Also note that:

$$\begin{aligned}
-\frac{\alpha^2}{2}\mathbf{Sat}^T(B\tilde{\boldsymbol{q}})M(\boldsymbol{q})\mathbf{Sat}(B\tilde{\boldsymbol{q}}) &\ge -\sum_{i=1}^n \frac{\alpha^2}{2}\lambda_{\max}\{M(\boldsymbol{q})\}\mathrm{Sat}^2(\beta_i\tilde{q};l,m) \\
&\ge -\sum_{i=1}^n \frac{\alpha^2}{2}\lambda_{\max}\{M(\boldsymbol{q})\}H(\tilde{q}_i) \\
H(\tilde{q}_i) &= \begin{cases} \beta_i^2\tilde{q}_i^2, & |\beta_i\tilde{q}_i| \le m \\ \varepsilon^2, & |\beta_i\tilde{q}_i| > m \end{cases} \quad (37)
\end{aligned}$$

From (36) and (37) we can conclude that there always exist large enough positive constants $\overline{k_{p_i}}$, $i = 1, ..., n$, such that:

$$G_i(\tilde{q}_i) > \frac{\alpha^2}{2}\lambda_{\max}\{M(\boldsymbol{q})\}H(\tilde{q}_i), \quad \forall \tilde{q}_i \ne 0 \in \mathbb{R}, \quad i = 1, ..., n \quad (38)$$

and hence:

$$\int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}} \mathbf{Sat}^T(B\boldsymbol{r})\overline{K_p}dr - \frac{\alpha^2}{2}\mathbf{Sat}^T(B\tilde{\boldsymbol{q}})M(\boldsymbol{q})\mathbf{Sat}(B\tilde{\boldsymbol{q}}) > 0 \quad \forall\, \tilde{\boldsymbol{q}} \ne \mathbf{0} \in \mathbb{R}^n \quad (39)$$

Now, we show that $\int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}}[-\boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{r}) + K_p^*\mathbf{Sat}(B\boldsymbol{r};\boldsymbol{l},\boldsymbol{m}) + \boldsymbol{g}(\boldsymbol{q}_d)]^T d\boldsymbol{r}$ is a positive definite function. First assume that $|\tilde{q}_i| \le l/\beta_i$, for $i = 1, ..., n$. Hence, according to Definition 4 we can write: $\int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}}[-\boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{r}) + K_p^*\mathbf{Sat}(B\boldsymbol{r};\boldsymbol{l}_i,\boldsymbol{m}_i) + \boldsymbol{g}(\boldsymbol{q}_d)]^T d\boldsymbol{r} = \int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}}[-\boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{r}) + K_p^*B\boldsymbol{r} + \boldsymbol{g}(\boldsymbol{q}_d)]^T d\boldsymbol{r}$. This mean that:

$$\frac{\partial}{\partial\tilde{\boldsymbol{q}}}\left[\frac{\partial}{\partial\tilde{\boldsymbol{q}}}\int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}}[-\boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{r}) + K_p^*B\boldsymbol{r} + \boldsymbol{g}(\boldsymbol{q}_d)]^T d\boldsymbol{r}\right] = \frac{\partial g(\boldsymbol{q})}{\partial\boldsymbol{q}} + K_p^*B, \quad \forall\ |\tilde{q}_i| \le l/\beta_i, \quad i = 1, ..., n \quad (40)$$

which, according to (Hernandez-Guzman and Silva-Ortigoza, 2011) and (Hernandez-Guzman *et al.*, 2008), is a positive definite if:

$$k_{p_i}^*\beta_i > k_{g_i} > \sum_{j=1}^{n} \max_{\boldsymbol{q}} \left| \frac{\partial g_i(\boldsymbol{q})}{\partial q_j} \right|, \quad i = 1, ..., n. \quad (41)$$

and, in base to Theorem 1, this proves that $\int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}}[-\boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{r}) + K_p^* B\boldsymbol{r} + \boldsymbol{g}(\boldsymbol{q}_d)]^T d\boldsymbol{r}$ is positive definite as long as $|\tilde{q}_i| \leq l/\beta_i$, for $i = 1, ..., n$. Now suppose that k and j represent all integers from 1 to n such that $\tilde{q}_j < l/\beta_j$ and $\tilde{q}_k < -l/\beta_k$. According to (Zavala-Rio and Santibañez, 2007), Definition 4 and Property 2, we can write:

$$\begin{aligned}
&\int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}}[-\boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{r}) + K_p^* \mathbf{Sat}(B\boldsymbol{r}; \boldsymbol{l}, \boldsymbol{m}) + \boldsymbol{g}(\boldsymbol{q}_d)]^T d\boldsymbol{r} \\
&= \sum_{i=1}^{n} \int_0^{\tilde{q}_i} [-\overline{g}_i(r_i) + g_i(\boldsymbol{q}_d) + k_{p_i}^* \text{Sat}(\beta_i r_i; l, m)] dr_i \\
&= P1 + P2 \qquad (42)
\end{aligned}$$

where

$$\begin{aligned}
\overline{g}_1(r_1) &= g_1(q_{d1} - r_1, q_{d_2}, ..., q_{d_n}) \\
\overline{g}_2(r_2) &= g_2(q_1, q_{d_2} - r_2, q_{d_3}, ..., q_{d_n}) \\
&\vdots \\
\overline{g}_n(r_n) &= g_n(q_1, q_2, ..., q_{n-1}, q_{d_n} - r_n)
\end{aligned}$$

and

$$\begin{aligned}
P1 &= \sum_{i=1, i\neq j \neq k}^{n} \int_0^{\tilde{q}_i} [-\overline{g}_i(r_i) + g_i(\boldsymbol{q}_d) + k_{p_i}^* \beta_i r_i] dr_i \\
&+ \sum_j \int_0^{l/\beta_j} [-\overline{g}_j(r_j) + g_j(\boldsymbol{q}_d) + k_{p_j}^* \beta_j r_j] dr_j \\
&+ \sum_k \int_0^{l/\beta_k} [-\overline{g}_k(r_k) + g_k(\boldsymbol{q}_d) + k_{p_k}^* \beta_k r_k] dr_k \\
P2 &= \sum_j \int_{l/\beta_j}^{\tilde{q}_j} [-\overline{g}_j(r_j) + g_j(\boldsymbol{q}_d) + k_{p_j}^* \text{Sat}(\beta_j r_j; l, m)] dr_j \\
&+ \sum_k \int_{-l/\beta_k}^{\tilde{q}_k} [-\overline{g}_k(r_k) + g_k(\boldsymbol{q}_d) + k_{p_k}^* \text{Sat}(\beta_k r_k; l, m)] dr_k \\
&\geq \sum_j \int_{l/\beta_j}^{\tilde{q}_j} [k_{p_j}^* l - 2k_j'] dr_j + \sum_k \int_{\tilde{q}_k}^{-l/\beta_k} [k_{p_k}^* l - 2k_k'] dr_k \\
&= \sum_j \left(k_{p_j}^* l - 2k_j'\right)\left(\tilde{q}_j - \frac{l}{\beta_j}\right) + \sum_k \left(k_{p_k}^* l - 2k_k'\right)\left(-\tilde{q}_k - \frac{l}{\beta_k}\right) \qquad (43)
\end{aligned}$$

Note that P_2 is always positive if:

$$k_{p_j}^* l > 2k_i', \quad (44)$$

because $\tilde{q}_j > l$ and $\tilde{q}_k < -l$. Finally, since we have proven that P_1 is positive we conclude that $\int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}}[-\boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{r}) + K_p^* \mathbf{Sat}(B\boldsymbol{r}; \boldsymbol{l}, \boldsymbol{m}) + \boldsymbol{g}(\boldsymbol{q}_d)]^T d\boldsymbol{r}$ is a positive definite and radially unbounded function of $\tilde{\boldsymbol{q}}$.

The latter prove that (34) is positive definite and radially unbounded.

It is possible to verify that, using Property 1, the time derivative of (34) along the trajectories of the closed-loop system (31) is given by:

$$\begin{aligned}
\dot{V} &= -\dot{\boldsymbol{q}}^T F_v \dot{\boldsymbol{q}} - \alpha \dot{\mathbf{Sat}}(B\tilde{\boldsymbol{q}}) M(\boldsymbol{q}) \dot{\boldsymbol{q}} - \alpha \mathbf{Sat}(B\tilde{\boldsymbol{q}})^T C(\boldsymbol{q}, \dot{\boldsymbol{q}})^T \dot{\boldsymbol{q}} \\
&- \alpha \mathbf{Sat}(B\tilde{\boldsymbol{q}})^T [\boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{g}(\boldsymbol{q})] - \alpha \mathbf{Sat}(B\tilde{\boldsymbol{q}})^T K_p \mathbf{Sat}(B\tilde{\boldsymbol{q}}) - \dot{\boldsymbol{q}}^T K_v \dot{\boldsymbol{q}} \qquad (45)
\end{aligned}$$

Negative semidefiniteness may be proved by upper bounding each element of (45):

$$\begin{aligned}
-\dot{\boldsymbol{q}}^T F_v \dot{\boldsymbol{q}} &\leq -\lambda_{\min}\{F_v\} \|\dot{\boldsymbol{q}}\|^2 \\
-\dot{\boldsymbol{q}}^T K_v \dot{\boldsymbol{q}} &\leq -\lambda_{\min}\{K_v\} \|\dot{\boldsymbol{q}}\|^2 \\
-\alpha \mathbf{Sat}(B\tilde{\boldsymbol{q}})^T C(\boldsymbol{q}, \dot{\boldsymbol{q}})^T \dot{\boldsymbol{q}} &\leq \alpha k_{c_1} m \sqrt{n} \|\dot{\boldsymbol{q}}\|^2 \\
-\alpha \dot{\mathbf{Sat}}(B\tilde{\boldsymbol{q}})^T M(\boldsymbol{q}) \dot{\boldsymbol{q}} &\leq \alpha \lambda_{\max}\{B\} \lambda_{\max}\{M\} \|\dot{\boldsymbol{q}}\|^2 \\
-\alpha \mathbf{Sat}(B\tilde{\boldsymbol{q}})^T K_p \mathbf{Sat}(B\tilde{\boldsymbol{q}}) &\leq -\alpha \sum_{i=1}^{n} k_{p_i} |\text{Sat}(\beta_i \tilde{q}_i)|^2 \\
-\alpha \mathbf{Sat}(B\tilde{\boldsymbol{q}})^T [\boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{g}(\boldsymbol{q})] &\leq \alpha \boldsymbol{h}^T (B\tilde{\boldsymbol{q}}) E \boldsymbol{h}(B\tilde{\boldsymbol{q}}) \qquad (46)
\end{aligned}$$

where $\beta_i \geq 1$, $i = 1, 2, ..., n$, $\boldsymbol{h}(B\tilde{\boldsymbol{q}}) = [|\text{Sat}(\beta_1 \tilde{q}_1)| ... |\text{Sat}(\beta_n \tilde{q}_n)|]^T$, and E is a symmetric matrix with all of its entries bounded (Hernandez-Guzman and Silva-Ortigoza, 2011). Finally the time derivative $\dot{V}$ can be upper bounded by:

$$\begin{aligned}
\dot{V} &\leq -[\lambda_{\min}\{F_v\} + \lambda_{\min}\{K_v\} - \alpha(m\sqrt{n} k_c + \lambda_{\max}\{B\} \lambda_{\max}\{M\})] \|\dot{\boldsymbol{q}}\|^2 \\
&- \alpha \sum_{i=1}^{n} [\lambda_{\min}\{K_p - E\}] \, |\text{Sat}(B\tilde{q}_i)|^2 \qquad (47)
\end{aligned}$$

By choosing K_p such that

$$\lambda_{\min}(K_p - E) > 0 \quad (48)$$

and a small enough value for $\alpha > 0$ such that

$$\begin{aligned}
&[\lambda_{\min}\{F_v\} + \lambda_{\min}\{K_v\} \\
&- \alpha(m\sqrt{n} k_{c_1} + \lambda_{\max}\{B\} \lambda_{\max}\{M\})] > 0, \qquad (49)
\end{aligned}$$

we have that (45) is negative semidefinite. Therefore, we can use the LaSalle invariance principle to ensure global asymptotic stability provided that (41), (44), (48) and (49) are satisfied. This complete the proof of Proposition 2.

IV. Experimental results

In this section is presented a experimental essays on the CICESE robot, using the controllers (13) and (30). The CICESE is a 2-dof robot manipulator with revolute joints whose dynamic model is presented in (Kelly and Santibañez, 2003). The numerical values of the parameters for the CICESE robot are shown in Table I. $\lambda_{\max}\{M\}$ and k_{c_1} were calculated following the procedure proposed in (Kelly and Santibañez, 2003).

TABLE I
NUMERICAL VALUES OF THE PARAMETERS FOR THE CICESE ROBOT.

Parameter	Joint 1	Joint 2	Units
k_{g_i}	42.1198	3.6540	[N m/rad]
k_i'	40.2928	1.8270	[N m]
$\tau_{\max}$	150	15	[N m]
$\lambda_{\max}\{M\}$	2.533		[kg m^2]
k_{c1}	0.336		[kg m^2]

In the following, the experimental results for each control law presented above are shown. The responses obtained in the experimental results is the best obtained for their respective controllers.

1. Consider the control law (13)

$$\begin{aligned}
\boldsymbol{\tau} &= K_p \tilde{\boldsymbol{q}} - K_v \dot{\boldsymbol{q}} + K_i \boldsymbol{w} \\
\boldsymbol{w} &= \int_0^t [\alpha \mathbf{sat}(\tilde{\boldsymbol{q}}(\sigma)) - \dot{\boldsymbol{q}}(\sigma)] d\sigma + \boldsymbol{w}(0)
\end{aligned}$$

which we have used to perform a real-time experimental essay on the CICESE robot. The controller gains used are presented in Table II. The joint errors and torques obtained in the experiment essay are shown in Fig. 1.

TABLE II
CONTROLLER GAINS.

gains	Joint 1	Joint 2	Units
K_p	95	9.5	[N m/rad]
K_v	20	16	[N m s/rad]
K_i	50	0.13	[N m / rad]
α	5		[1/s]
L, M	1.1, 1.2		[rad]

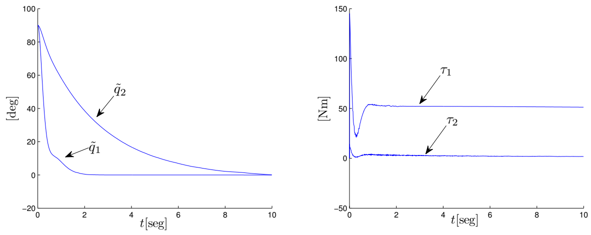

Figura 1. Position errors and torque obtained using control law (13).

2. Consider the control law (30)

$$\boldsymbol{\tau} = K_p \mathbf{Sat}(B\tilde{\boldsymbol{q}}; l, m) - K_v \dot{\boldsymbol{q}} + K_i \boldsymbol{w}$$
$$\boldsymbol{w} = \int_0^t [\alpha \mathbf{Sat}(B\tilde{\boldsymbol{q}}(\sigma); l, m) - \dot{\boldsymbol{q}}(\sigma)] d\sigma + \boldsymbol{w}(0),$$

which we have used to perform a real-time experimental essay on the CICESE robot. The controller gains used are presented in Table III. The joint errors and torques obtained in the experiment essay are shown in Fig. 2.

TABLE III
CONTROLLER GAINS.

gains	Joint 1	Joint 2	Units
K_p	125	12.5	[N m/rad]
K_v	50	4	[N m s/rad]
K_i	1	1	[N m / rad]
B	30	20	[N m / rad]
α	0.000001		[1/s]
L, M	1.1, 1.2		[rad]

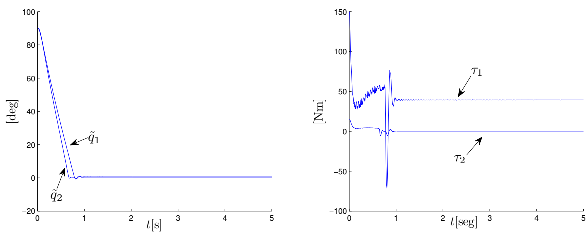

Figura 2. Position errors and torque obtained using control law (30).

The controller parameters for both controllers were selected in such a way that the torque of each actuator remains under the maximum permitted torque ($|\tau_i| < \tau_{\max_i}$). Due to the control law (30) has a saturation function in the proportional part, it is possible choose larger proportional gains than those of the control law (13). This fact allows to achieve the desired position faster for the controller (30) than the controller (13) (see Fig. 1 and 2). It is easy to prove that controller parameters fulfill the tuning conditions for assuring global asymptotic stability.

V. CONCLUSIONS

In this paper new tuning conditions for a class of nonlinear PID global regulators were found. By using Lyapunov's stability theory new tuning conditions were established, which are less restrictive than those shown in (Santibañez and Kelly, 1998), (Arimoto, 1995) and (Kelly, 1998). For validation purposes, experimental results were reported using the CICESE robot manipulator.

VI. ACKNOWLEDGMENTS

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